

CHAPTER VII., THE PYTHAGOREANS

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138. The Pythagorean School

AFTER losing their supremacy in the Achaian cities, the Pythagoreans concentrated themselves at Rhegion; but the school founded there did not maintain itself for long, and only Archytas stayed behind in Italy. Philolaos and Lysis, the latter of whom had escaped as a young man from the massacre of Kroton, had already found their way to Thebes.¹ We know from Plato that Philolaos was there towards the close of the fifth century, and Lysis was afterwards the teacher of Epameinondas.² Some of the Pythagoreans, however, were able to return to Italy later. Philolaos certainly did so, and Plato implies that he had left Thebes some time before 399 B.C., the year Sokrates was put to death. In the fourth century, the chief seat of the school is the Dorian city of Taras, and we find the Pythagoreans heading the opposition to Dionysios of Syracuse. It is to this period that the activity of Archytas belongs. He was the friend of Plato, and almost realised the ideal of the philosopher king. He ruled Taras for years, and Aristoxenos tells us that he was never defeated in the field of battle.³ He was also the inventor of mathematical mechanics. At the same time, Pythagoreanism had taken root in the East. Lysis remained at Thebes, where Simmias and Kebes had heard Philolaos, while the remnant of the

Pythagorean school of Rhegion settled at Phleious. Aristoxenos was personally acquainted with the last generation of this school, and mentioned by name Xenophilos the Chalkidian from Thrace, with Phanton, Echekrates, Diokles, and Polymnastos of Phleious. They were all, he said, disciples of Philolaos and Eurytos,⁴ and we learn from Plato that Simmias and Kebes of Thebes and Echekrates of Phleious were also associates of Sokrates.⁵ Xenophilos was the teacher of Aristoxenos, and lived in perfect health at Athens to the age of a hundred and five.⁶

139. Philolaos

This generation of the school really belongs, however, to a later period; it is with Philolaos we have now to deal. The facts we know about his teaching from external sources are few in number. The doxographers, indeed, ascribe to him an elaborate theory of the planetary system, but Aristotle never mentions his name in connexion with that. He gives it as the theory of "the Pythagoreans" or of "some Pythagoreans."⁷ It seems natural to suppose, however, that the Pythagorean elements of Plato's *Phaedo* and *Gorgias* come mainly from Philolaos. Plato makes Sokrates express surprise that Simmias and Kebes had not learnt from him why it is unlawful for a man to take his life,⁸ and it seems to be implied that the Pythagoreans at Thebes used the word "philosopher" in the special sense of a man who is seeking to find a way of release from the burden of this life.⁹ It is probable that Philolaos spoke of the body ($\sigma\tilde{\omega}\mu\alpha$) as the tomb ($\sigma\tilde{\eta}\mu\alpha$) of the soul.¹⁰ We seem to be justified, then, in holding that he taught the old Pythagorean religious doctrine in some form, and that he laid special stress on knowledge as a means of release. That is the impression we get from Plato, who is far the best authority we have.

We know further that Philolaos wrote on "numbers"; for Speusippos followed him in the account he gave of the Pythagorean theories on that subject.¹¹ It is probable that he busied himself mainly with arithmetic, and we can hardly doubt that his geometry was of the primitive type described in an earlier chapter. Eurytos was his disciple, and we have seen (§ 47) that his views were still very crude.

We also know now that Philolaos wrote on medicine,¹² and that, while apparently influenced by the theories of the Sicilian school, he opposed them from the Pythagorean standpoint. In particular, he said that our bodies were composed only of the warm, and did not participate in the cold. It was only after birth that the cold was introduced by respiration. The connexion of this with the old Pythagorean theory is clear. Just as the Fire in the macrocosm draws in and limits the cold dark breath which surrounds the world (§ 53), so do our bodies inhale cold breath from outside. Philolaos made bile, blood, and phlegm the causes of disease; and, in accordance with this theory, he had to deny that the phlegm was cold, as the Sicilian school held. Its etymology proved it to be warm. We shall see that it

was probably this preoccupation with the medicine of the Sicilian school that gave rise to some of the most striking developments of later Pythagoreanism.

140. Plato and the Pythagoreans

Such, so far as I can judge, was the historical Philolaos, though he is usually represented in a very different light and has even been called a predecessor of Copernicus. To understand this, we must turn our attention to the story of a literary conspiracy.

We have seen that there are one or two references to Philolaos in Plato,¹³ but these hardly suggest that he played an important part in the development of Pythagorean science. The most elaborate account we have of this is put by Plato into the mouth of Timaios the Lokrian, of whom we know no more than he has chosen to tell us. It is clear at least that he is supposed to have visited Athens when Sokrates was still in the prime of life,¹⁴ and that he must have been practically a contemporary of Philolaos. It hardly seems likely that Plato should have given him the credit of discoveries which were really due to his better known contemporary. However, Plato had many enemies and detractors, and Aristoxenos was one of them. We know he made the extraordinary statement that most of the *Republic* was to be found in a work by Protagoras,¹⁵ and he seems also to be the original source of the story that Plato bought "three Pythagorean books" from Philolaos and copied the *Timaeus* out of them. According to this, the "three books" had come into the possession of Philolaos; and, as he had fallen into great poverty, Dion was able to buy them from him, or from his relatives, at Plato's request, for a hundred *minae*.¹⁶ It is certain, at any rate, that this story was already current in the third century; for the sillographer Timon of Phleious addresses Plato thus: "And of thee too, Plato, did the desire of discipleship lay hold. For many pieces of silver thou didst get in exchange a small book, and starting from it didst learn to write *Timaeus*."¹⁷ Hermippos, the pupil of Kallimachos, said that "some writer" said Plato himself bought the books from the relatives of Philolaos for forty Alexandrian *minae* and copied the *Timaeus* out of it; while Satyros, the Aristarchean, says he got it through Dion for a hundred *minae*.¹⁸ There is no suggestion in any of these accounts that the book was by Philolaos himself; they imply rather that what Plato bought was either a book by Pythagoras, or at any rate authentic notes of his teaching, which had come into the hands of Philolaos. In later times, it was generally supposed that the forgery entitled *The Soul of the World*, which goes by the name of Timaios the Lokrian, was meant;¹⁹ but it has now been proved that this cannot have existed earlier than the first century A.D. Moreover, it is plain that it is based on Plato's *Timaeus* itself, and that it was written in order to bolster up the story of Plato's plagiarism. It does not, however, fulfil the most important requirement, that of being in three books, which is always an essential feature of that story.²⁰

Not one of the writers just mentioned professes to have seen these famous "three books";²¹ but at a later date there were at least two works which claimed to represent them. Diels has shown how a treatise in three sections, entitled Παιδευτικόν, πολιτικόν, φυσικόν, was composed in the Ionic dialect and attributed to Pythagoras. It was largely based on the Πυθαγορικὰ ἀποφάσεις of Aristoxenos, but its date is uncertain.²² In the first century B.C., Demetrios Magnes professes to quote the opening words of the work published by Philolaos.²³ These, however, are in Doric. Demetrios does not actually say this work was written by Philolaos himself, though it is no doubt the same from which a number of extracts are preserved under his name in Stobaios and later writers. If it professed to be by Philolaos, that was not quite in accordance with the original story; but it is easy to see how his name may have become attached to it. We are told that the other book which passed under the name of Pythagoras was really by Lysis.²⁴ Boeckh has shown that the work ascribed to Philolaos probably consisted of three books also, and Proclus referred to it as the *Bakchai*,²⁵ a fanciful Alexandrian title which recalls the "Muses" of Herodotos. Two of the extracts in Stobaios bear it. It must surely be confessed that the whole story is very suspicious.

141. The "Fragments of Philolaos"

Boeckh argued that all the fragments preserved under the name of Philolaos were genuine; but no one will now go so far as that. The lengthy extract on the soul is given up even by those who maintain the genuineness of the rest.²⁶ It cannot be said that this position is plausible. Boeckh saw there was no ground for supposing that there ever was more than a single work, and he drew the conclusion that we must accept all the remains as genuine or reject all as spurious.²⁷ As, however, many scholars still maintain the genuineness of most of the fragments, we cannot ignore them altogether. Arguments based on their doctrine would, it is true, present the appearance of a vicious circle at this stage, but there are two serious objections to the fragments, which may be mentioned at once.

In the first place, we must ask whether it is likely that Philolaos should have written in Doric? Ionic was the dialect of science and philosophy till the time of the Peloponnesian War, and there is no reason to suppose the early Pythagoreans used any other.²⁸ Pythagoras was himself an Ionian, and it is not likely that in his time the Achaian states in which he founded his Order had adopted the Dorian dialect.²⁹ Alkmaion of Kroton seems to have written in Ionic.³⁰ Diels says that Philolaos and then Archytas were the first Pythagoreans to use the dialect of their homes;³¹ but Philolaos can hardly be said to have had a home, and it is hard to see why an Achaian refugee at Thebes should write in Doric.³² Nor did Archytas write in the Laconian dialect of Taras, but in what may be called "common Doric," and he is a generation later than Philolaos, which makes a great difference. In the time of Philolaos and later, Ionic was still used even by the citizens of Dorian states for scientific purposes. The

Syracusan historian Antiochos wrote in Ionic, and so did the medical writers of Dorian Kos and Knidos. The forged work of Pythagoras, which some ascribed to Lysis, was in Ionic; and so was the book on the *Akousmata* attributed to Androkydes,³³ which shows that, even in Alexandrian times, it was believed that Ionic was the proper dialect for Pythagorean writings.

In the second place, there can be no doubt that one of the fragments refers to the five regular solids, four of which are identified with the elements of Empedokles.³⁴ Now Plato tells us in the *Republic* that stereometry had not been adequately investigated at the time that dialogue is supposed to take place,³⁵ and we have express testimony that the five "Platonic figures," as they were called, were discovered in the Academy. In the Scholia to Euclid we read that the Pythagoreans only knew the cube, the pyramid (tetrahedron), and the dodecahedron, while the octahedron and the icosahedron were discovered by Theaitetos.³⁶ This sufficiently justifies us in regarding the "fragments of Philolaos" with suspicion, and all the more so as Aristotle does not appear to have seen the work from which these fragments come.³⁷

142. The Problem

We must look, then, for other evidence. From what has been said, it will be clear that it is above all from Plato we can learn to regard Pythagoreanism sympathetically. Aristotle was out of sympathy with Pythagorean ways of thinking, but he took great pains to understand them. This was because they played so great a part in the philosophy of Plato and his successors, and he had to make the relation of the two doctrines as clear as he could to himself and his disciples. What we have to do, then, is to interpret what Aristotle tells us in the spirit of Plato, and then to consider how the doctrine we thus arrive at is related to the systems which preceded it. It is a delicate operation, no doubt, but it has been made much safer by recent discoveries in the early history of mathematics and medicine.

Zeller has cleared the ground by eliminating the Platonic elements which have crept into later accounts of the system. These are of two kinds. First of all, we have genuine Academic formulae, such as the identification of the Limit and the Unlimited with the One and the Indeterminate Dyad;³⁸ and secondly, there is the Neoplatonic doctrine which represents the opposition between them as one between God and Matter.³⁹ It is not necessary to repeat Zeller's arguments here, as no one will now attribute the doctrine in that form to the Pythagoreans.

This simplifies the problem, but it is still very difficult. According to Aristotle, the Pythagoreans said Things are numbers, though that is not the doctrine of the fragments of "Philolaos." According to them, things have number, which makes them knowable, while their real essence is something unknowable.⁴⁰ We have seen reason for believing that Pythagoras himself said *Things are numbers* (§ 52),

and there is no doubt as to what his followers meant by the formula; for Aristotle says they used it in a cosmological sense. The world, according to them, was made of numbers in the same sense as others had said it was made of "four roots" or "innumerable seeds." It will not do to dismiss this as mysticism. The Pythagoreans of the fifth century were scientific men, and must have meant something quite definite. We shall, no doubt, have to say that they used the words *Things are numbers* in a somewhat non-natural sense, but there is no difficulty in that. The Pythagoreans had a great veneration for the actual words of the Master (ἀὐτῷ ἔφα); but such veneration is often accompanied by a singular licence of interpretation. We shall start, then, from what Aristotle tells us about the numbers.

143. Aristotle on the Numbers

In the first place, Aristotle is quite clear that Pythagoreanism was intended to be a cosmological system like the others. "Though the Pythagoreans," he tells us, "made use of less obvious first principles and elements than the rest, seeing that they did not derive them from sensible objects, yet all their discussions and studies had reference to nature alone. They describe the origin of the heavens, and they observe the phenomena of its parts, all that happens to it and all it does."⁴¹ They apply their first principles entirely to these things, "agreeing apparently with the other natural philosophers in holding that reality was just what could be perceived by the senses, and is contained within the compass of the heavens,"⁴² though "the first principles and causes they made use of were really adequate to explain realities of a higher order than the sensible."⁴³

The doctrine is more precisely stated by Aristotle to be that the elements of numbers are the elements of things, and that therefore things are numbers.⁴⁴ He is equally positive that these "things" are sensible things,⁴⁵ and indeed that they are bodies,⁴⁶ the bodies of which the world is constructed.⁴⁷ This construction of the world out of numbers was a real process in time, which the Pythagoreans described in detail.⁴⁸

Further, the numbers were intended to be mathematical numbers, though they were not separated from the things of sense.⁴⁹ On the other hand, they were not mere predicates of something else, but had an independent reality of their own. "They did not hold that the limited and the unlimited and the one were certain other substances, such as fire, water, or anything else of that sort; but that the unlimited itself and the one itself were the reality of the things of which they are predicated, and that is why they said that number was the reality of everything."⁵⁰ Accordingly the numbers are, in Aristotle's own language, not only the formal, but also the material, cause of things.⁵¹

Lastly, Aristotle notes that the point in which the Pythagoreans agreed with Plato was in giving numbers an independent reality of their own; while Plato differed from the Pythagoreans in holding

that this reality was distinguishable from that of sensible things.⁵² Let us consider these statements in detail.

144. The Elements of Numbers

Aristotle speaks of certain "elements" (στοιχεῖα) of numbers, which were also the elements of things. That is clearly the key to the problem, if we can discover what it means. Primarily, the "elements of number" are the Odd and the Even, but that does not seem to help us much. We find, however, that the Odd and Even were identified with the Limit and the Unlimited, which we have seen reason to regard as the original principles of the Pythagorean cosmology (§ 53). Aristotle tells us that it is the Even which gives things their unlimited character when it is contained in them and limited by the Odd,⁵³ and the commentators are at one in understanding this to mean that the Even is in some way the cause of infinite divisibility. They get into difficulties, however, when they try to show how this can be. Simplicius has preserved an explanation, in all probability Alexander's, to the effect that they called the even number unlimited "because every even is divided into equal parts, and what is divided into equal parts is unlimited in respect of bipartition; for division into equals and halves goes on *ad infinitum*. But, when the odd is added, it limits it; for it prevents its division into equal parts."⁵⁴ Now it is plain that we must not impute to the Pythagoreans the view that even numbers can be halved indefinitely. They must have known that the even numbers 6 and 10 can only be halved once. The explanation is rather to be found in a fragment of Aristoxenos, where we read that "even numbers are those which are divided, into equal parts, while odd numbers are divided into unequal parts and have a middle term."⁵⁵ This is still further elucidated by a passage which is quoted in Stobaios and ultimately goes back to Poseidonios. It runs: "When the odd is divided into two equal parts, a unit is left over in the middle; but when the even is so divided, an empty field is left, without a master and without a number, showing that it is defective and incomplete."⁵⁶ Again, Plutarch says: "In the division of numbers, the even, when parted in any direction, leaves as it were within itself . . . a field; but, when the same thing is done to the odd, there is always a middle left over from the division."⁵⁷ It is clear that all these passages refer to the same thing, and that can hardly be anything else than the "terms" or dots with which we are already familiar (§ 47). The division must fall between these; for, if it meets with an indivisible unit, it is at once arrested.

145. The Numbers Spatial

Now there can be no doubt that by his Unlimited Pythagoras meant something spatially extended; for he identified it with air, night, or the void. We are prepared, then, to find that his followers also thought of the Unlimited as extended. Aristotle certainly regarded it so. He argues that, if the Unlimited is itself a reality, and not merely the predicate of some other reality, then every part of it

must be unlimited too, just as every part of air is air.⁵⁸ The same thing is implied in his statement that the Pythagorean Unlimited was outside the heavens.⁵⁹ Further than this, it is not safe to go. Philolaos and his followers cannot have regarded the Unlimited as Air; for, as we shall see, they adopted the theory of Empedokles as to that "element," and accounted for it otherwise. One of them, Xouthos, argued that rarefaction and condensation implied the void; without it the universe would overflow.⁶⁰ We do not know, however, whether he was earlier than the Atomists or not. It is enough to say that the Pythagoreans meant by the Unlimited the *res extensa*.

As the Unlimited is spatial, the Limit must be spatial too, and we should expect to find that the point, the line, and the surface were regarded as forms of the Limit. That was the later doctrine; but the characteristic feature of Pythagoreanism is just that the point was not regarded as a limit, but as the first product of the Limit and the Unlimited, and was identified with the arithmetical unit instead of with zero. According to this view, then, the point has one dimension, the line two, the surface three, and the solid four.⁶¹ In other words, the Pythagorean points have magnitude, their lines breadth, and their surfaces thickness. The whole theory, in short, turns on the definition of the point as a unit "having position" (*μονὰς θέσιν ἔχουσα*).⁶² It was out of such elements that it seemed possible to construct a world.

146. The Numbers as Magnitudes

This way of regarding the point, the line, and the surface is closely bound up with the practice of representing numbers by dots arranged in symmetrical patterns, which we have seen reason for attributing to the Pythagoreans (§ 47). Geometry had already made considerable advances, but the old view of quantity as a sum of units had not been revised, and so, the point was identified with 1 instead of with 0. That is the answer to Zeller's contention that to regard the Pythagorean numbers as spatial is to ignore the fact that the doctrine was originally arithmetical rather than geometrical. Our interpretation takes full account of that fact, and indeed makes the peculiarities of the whole system depend on it. Aristotle is very decided as to the Pythagorean points having magnitude. "They construct the whole world out of numbers," he tells us, "but they suppose the units have magnitude. As to how the first unit with magnitude arose, they appear to be at a loss."⁶³ Zeller holds that this is only an inference of Aristotle's,⁶⁴ and he is probably right in this sense, that the Pythagoreans never felt the need of saying in so many words that points had magnitude. It does seem probable, however, that they called them *ὄγκοι*.⁶⁵

Zeller, moreover, allows, and indeed insists, that in the Pythagorean cosmology the numbers were spatial, but he raises difficulties about the other parts of the system. There are other things, such as the Soul and Justice and Opportunity, which are said to be numbers, and which cannot be regarded

as constructed of points, lines, and surfaces.⁶⁶ Now it appears to me that this is just the meaning of a passage in which Aristotle criticises the Pythagoreans. They held, he says, that in one part of the world Opinion prevailed, while a little above it or below it were to be found Injustice or Separation or Mixture, each of which was, according to them, a number. But in the very same regions of the heavens were to be found things having magnitude which were also numbers. How can this be, since justice has no magnitude?⁶⁷ This means surely that the Pythagoreans had failed to give any clear account of the relation between these more or less fanciful analogies and their geometrical construction of the universe.

147. The Numbers and the Elements

We seem to see further that what distinguished the Pythagoreanism of this period from its earlier form was that it sought to adapt itself to the new theory of "elements." This is what makes it necessary to take up the consideration of the system once more in connexion with the pluralists. When the Pythagoreans returned to Southern Italy, they would find views prevalent there which demanded a partial reconstruction of their own system. We do not know that Empedokles founded a philosophical society, but there can be no doubt of his influence on the medical school of these regions; and we also know now that Philolaos played a part in the history of medicine.⁶⁸ This gives us the clue to what formerly seemed obscure. The tradition is that the Pythagoreans explained the elements as built up of geometrical figures, a theory we can study for ourselves in the more developed form it attained in Plato's *Timaeus*.⁶⁹ If they were to retain their position as the leaders of medical study in Italy, they were bound to account for the elements.

We must not take it for granted, however, that the Pythagorean construction of the elements was exactly the same as that we find in Plato's *Timaeus*. As we have seen, there is good reason for believing they only knew three of the regular solids, the cube, the pyramid (tetrahedron), and the dodecahedron.⁷⁰ Now Plato makes Timaios start from fire and earth,⁷¹ and in the construction of the elements he proceeds in such a way that the octahedron and the icosahedron can easily be transformed into pyramids, while the cube and the dodecahedron cannot. From this it follows that, while air and water pass readily into fire, earth cannot do so,⁷² and the dodecahedron is reserved for another purpose, which we shall consider presently. This would exactly suit the Pythagorean system; for it would leave room for a dualism of the kind outlined in the Second Part of the poem of Parmenides. We know that Hippasos made Fire the first principle, and we see from the *Timaeus* how it would be possible to represent air and water as forms of fire. The other element is, however, earth, not air, as we have seen reason to believe that it was in early Pythagoreanism. That would be a natural result of the discovery of atmospheric air by Empedokles and of his general theory of the elements. It would also explain the

puzzling fact, which we had to leave unexplained above, that Aristotle identifies the two "forms" spoken of by Parmenides with Fire and Earth.⁷³

148. The Dodecahedron

The most interesting point in the theory is, however, the use made of the dodecahedron. It was identified, we are told, with the "sphere of the universe," or, as it is put in the Philolaic fragment, with the "hull of the sphere."⁷⁴ Whatever we may think of the authenticity of the fragments there is no reason to doubt that this is a genuine Pythagorean expression, and it must be taken in close connexion with the word "keel" applied to the central fire.⁷⁵ The structure of the world was compared to the building of a ship, an idea of which there are other traces.⁷⁶ The key to what we are told of the dodecahedron is also given by Plato. In the *Phaedo*, which must have been written before the doctrine of the regular solids was fully established, we read that the "true earth," if looked at from above, is "many-coloured like the balls that are made of twelve pieces of leather."⁷⁷ In the *Timaeus* the same thing is referred to in those words: "Further, as there is still one construction left, the fifth, God made use of it for the universe when he painted it."⁷⁸ The point is that the dodecahedron approaches more nearly to the sphere than any other of the regular solids. The twelve pieces of leather used to make a ball would all be regular pentagons; and, if the material were not flexible like leather, we should have a dodecahedron instead of a sphere. That proves that the dodecahedron was well known before Theaitetos, and we may infer that it was regarded as forming the "timbers" on which the spherical hulk of the heavens was built.

The tradition confirms in an interesting way the importance of the dodecahedron in the Pythagorean system. According to one account, Hippasos was drowned at sea for revealing "the sphere formed out of the twelve pentagons."⁷⁹ The Pythagorean construction of the dodecahedron we may partially infer from the fact that they adopted the pentagram or *pentalpha* as their symbol. The use of this figure in later magic is well known; and Paracelsus still employed it as a symbol of health, which is exactly what the Pythagoreans called it.⁸⁰

149. The Soul as Harmony

The view that the soul is a "harmony," or rather an attunement, is intimately connected with the theory of the four elements. It cannot have belonged to the earliest form of Pythagoreanism; for, as shown in Plato's *Phaedo*, it is quite inconsistent with the idea that the soul can exist independently of the body. It is the very opposite of the belief that "any soul can enter any body."⁸¹ On the other hand, we are told in the *Phaedo* that it was accepted by Simmias and Kebes, who had heard Philolaos at Thebes, and by Echekrates of Phleious, who was the disciple of Philolaos and Eurytos.⁸² The account of the

doctrine given by Plato is quite in accordance with the view that it was of medical origin. Simmias says: "Our body being, as it were, strung and held together by the warm and the cold, the dry and the moist, and things of that sort, our soul is a sort of temperament and attunement of these, when they are mingled with one another well and in due proportion. If, then, our soul is an attunement, it is clear that, when the body has been relaxed or strung up out of measure by diseases and other ills, the soul must necessarily perish at once."⁸³ This is clearly an application of the theory of Alkmaion (§ 96), and is in accordance with the views of the Sicilian school. It completes the evidence that the Pythagoreanism of the end of the fifth century was an adaptation of the old doctrine to the new principles introduced by Empedokles.

It is further to be observed that, if the soul is regarded as an attunement in the Pythagorean sense, we should expect it to contain the three intervals then recognised, the fourth, the fifth and the octave, and this makes it extremely probable that Poseidonios was right in saying that the doctrine of the tripartite soul, as we know it from the *Republic* of Plato, was really Pythagorean. It is quite inconsistent with Plato's own view of the soul, but agrees admirably with that just explained.⁸⁴

150. The Central Fire

The planetary system which Aristotle attributes to "the Pythagoreans" and Aetios to Philolaos is sufficiently remarkable.⁸⁵ The earth is no longer in the middle of the world; its place is taken by a central fire, which is not to be identified with the sun. Round this fire revolve ten bodies. First comes the *Antichthon* or Counter-earth, and next the earth, which thus becomes one of the planets. After the earth comes the moon, then the sun, the planets, and the heaven of the fixed stars. We do not see the central fire and the *antichthon* because the side of the earth on which we live is always turned away from them. This is to be explained by the analogy of the moon, which always presents the same face to us, so that men living on the other side of it would never see the earth. This implies, of course, from our point of view, that these bodies rotate on their axes in the same time as they revolve round the central fire,⁸⁶ and that the *antichthon* revolves round the central fire in the same time as the earth, so that it is always in opposition to it.⁸⁷

It is not easy to accept the statement of Aetios that this system was taught by Philolaos. Aristotle nowhere mentions him in connexion with it, and in the *Phaedo* Sokrates gives a description of the earth and its position in the world which is entirely opposed to it, but is accepted without demur by Simmias the disciple of Philolaos.⁸⁸ It is undoubtedly a Pythagorean theory, however, and marks a noticeable advance on the Ionian views current at Athens. It is clear too that Sokrates states it as something of a novelty that the earth does not require the support of air or anything of the sort to keep it in its place. Even Anaxagoras had not been able to shake himself free of that idea, and Demokritos

still held it along with the theory of a flat earth. The natural inference from the *Phaedo* would certainly be that the theory of a spherical earth, kept in the middle of the world by its equilibrium, was that of Philolaos himself. If so, the doctrine of the central fire would belong to a later generation.

It seems probable that the theory of the earth's revolution round the central fire really originated in the account of the sun's light given by Empedokles. The two things are brought into close connexion by Aetios, who says that Empedokles believed in two suns, while "Philolaos" believed in two or even in three. His words are obscure, but they seem to justify us in holding that Theophrastos regarded the theories as akin.⁸⁹ We saw that Empedokles gave two inconsistent explanations of the alternation of day and night (§ 113), and it may well have seemed that the solution of the difficulty was to make the sun shine by reflected light from a central fire. Such a theory would, in fact, be the natural issue of recent discoveries as to the moon's light and the cause of its eclipses, if these were extended to the sun, as they would almost inevitably be.

The central fire received a number of mythological names, such as the "hearth of the world," the "house," or "watch-tower" of Zeus, and "the mother of the gods."⁹⁰ That was in the manner of the school, but it must not blind us to the fact that we are dealing with a scientific hypothesis. It was a great thing to see that the phenomena could best be "saved" by a central luminary, and that the earth must therefore be a revolving sphere like the other planets.⁹¹ Indeed, we are tempted to say that the identification of the central fire with the sun was a detail in comparison. It is probable, at any rate, that this theory started the train of thought which made it possible for Aristarchos of Samos to reach the heliocentric hypothesis,⁹² and it was certainly Aristotle's successful reassertion of the geocentric theory which made it necessary for Copernicus to discover the truth afresh. We have his own word for it that he started from what he had read about the Pythagoreans.⁹³

In the form in which it was now stated, however, the theory raised almost as many difficulties as it solved, and it did not maintain itself for long. It is clear from Aristotle that its critics raised the objection that it failed to "save the phenomena" inasmuch as the assumed revolution of the earth would produce parallaxes too great to be negligible and that the Pythagoreans gave some reason for the belief that they were negligible. Aristotle has no clear account of the arguments on either side, but it may be pointed out that the earth was probably supposed to be far smaller than it is, and there is no reason why its orbit should have been thought to have an appreciably greater diameter than we now know the earth itself to have.⁹⁴

A truer view of the earth's dimensions would naturally suggest that the alternation of night and day was due to the earth's rotation on its own axis, and in that case the earth could once more be regarded as in the centre. It does not appear that Aristotle knew of any one who had held this view, but

Theophrastos seems to have attributed it to Hiketas and Ekphantos of Syracuse, of whom we know very little otherwise.⁹⁵ Apparently they regarded the heaven of the fixed stars as stationary, a thing Aristotle would almost have been bound to mention if he had ever heard of it, since his own system turns entirely on the diurnal revolution.

Both theories, that of the earth's revolution round a central fire and that of its rotation on its own axis, had the effect of making the revolution of the fixed stars, to which the Pythagoreans certainly adhered, very difficult to account for. They must either be stationary or their motion must be something quite different from the diurnal revolution.⁹⁶ It was probably this that led to the abandonment of the theory.

In discussing the views of those who hold the earth to be in motion, Aristotle only mentions one theory as alternative to that of its revolution round the central fire, and he says that it is that of the *Timaeus*. According to this the earth is not one of the planets but "at the centre," while at the same time it has some kind of motion relatively to the axis of the universe.⁹⁷ Now this motion can hardly be an axial rotation, as was held by Grote;⁹⁸ for the whole cosmology of the *Timaeus* implies that the alternation of day and night is due to the diurnal revolution of the heavens.⁹⁹ The fact that the earth is referred to a little later as "the guardian and artificer of night and day"¹⁰⁰ proves nothing to the contrary, since night is in any case the conical shadow of the earth, which is thus the cause of the alternation of day and night. So far, Boeckh and his followers appear to be in the right.

When, however, Boeckh goes on to argue that the word ἰλλομένην in the *Timaeus* does not refer to motion at all, but that it means "globed" or "packed" round, it is quite impossible for me to follow him. Apart from all philological considerations, this interpretation makes nonsense of Aristotle's line of argument. He says¹⁰¹ that, if the earth is in motion, whether "outside the centre" or "at the centre," that cannot be a "natural motion"; for, if it were, it would be shared by every particle of earth, and we see that the natural motion of every clod of earth is "down," *i.e.* towards the centre. He also says that, if the earth is in motion, whether "outside the centre" or "at the centre," it must have two motions like everything else but the "first sphere," and therefore there would be excursions in latitude (πάροδοι) and "turnings back" (τροπαί) of the fixed stars, which there are not. It is clear, then, that Aristotle regarded the second theory of the earth's movement as involving a motion of translation equally with the first, and that he supposed it to be the theory of Plato's *Timaeus*. It is impossible to believe that he can have been mistaken on such a point.¹⁰²

When we turn to the passage in the *Timaeus* itself, we find that, when the text is correctly established, it completely corroborates Aristotle's statement that a motion of translation is involved,¹⁰³

and that Boeckh's rendering is inadmissible on grammatical and lexicological grounds.¹⁰⁴ We have therefore to ask what motion of translation is compatible with the statement that the earth is "at the centre," and there seems to be nothing left but a motion up and down (to speak loosely) on the axis of the universe itself. Now the only clearly attested meaning of the rare word ἄλλομαι is just that of motion to and fro, backwards and forwards.¹⁰⁵ It may be added that a motion of this kind was familiar to the Pythagoreans, if we may judge from the description of the waters in the earth given by Sokrates in the *Phaedo*, on the authority of some unnamed cosmologist.¹⁰⁶

What was this motion intended to explain? It is impossible to be certain, but it is clear that the motions of the circles of the Same and the Other, *i.e.* the equator and the ecliptic, are inadequate to "save the appearances." So far as they go, all the planets should either move in the ecliptic or remain at an invariable distance from it, and this is far from being the case. Some explanation is required of their excursions in latitude, *i.e.* their alternate approaches to the ecliptic and departures from it. We have seen (p. 63) that Anaximander already busied himself with the "turnings back" of the moon. Moreover, the direct and retrograde movements of the planets are clearly referred to in the *Timaeus* a few lines below.¹⁰⁷ We are not bound to show in detail that a motion of the kind suggested would account for these apparent irregularities; it is enough if it can be made probable that the fifth-century Pythagoreans thought it could. It may have seemed worth while to them to explain the phenomena by a regular motion of the earth rather than by any waywardness in the planets; and, if so, they were at least on the right track.

To avoid misunderstanding, I would add that I do not suppose Plato himself was satisfied with the theory which he thought it appropriate for a Pythagorean of an earlier generation to propound. The idea that Plato expounded his own personal views in a dialogue obviously supposed to take place before he was born, is one which, to me at least, is quite incredible. We know, moreover, from the unimpeachable authority of Theophrastos, who was a member of the Academy in Plato's later years, that he had then abandoned the geocentric hypothesis, though we have no information as to what he supposed to be in the centre of our system.¹⁰⁸ It seems clear too from the *Laws* that he must have attributed an axial rotation to the earth.¹⁰⁹

151. The Antichthon

The existence of the *antichthon* was also a hypothesis intended to account for the phenomena of eclipses. In one place, indeed, Aristotle says the Pythagoreans invented it in order to bring the number of revolving bodies up to ten;¹¹⁰ but that is a mere sally, and Aristotle really knew better. In his work on the Pythagoreans, he said that eclipses of the moon were caused sometimes by the intervention of the earth and sometimes by that of the *antichthon*; and the same statement was made by Philip of Opous, a

very competent authority on the matter.¹¹¹ Indeed, Aristotle shows in another passage how the theory originated. He tells us that some thought there might be a considerable number of bodies revolving round the centre, though invisible to us because of the intervention of the earth, and that they accounted in this way for there being more eclipses of the moon than of the sun.¹¹² This is mentioned in close connexion with the *antichthon*, so Aristotle clearly regarded the two hypotheses as of the same nature. The history of the theory seems to be this. Anaximenes had assumed the existence of dark planets to account for lunar eclipses (§ 29), and Anaxagoras had revived that view (§ 135). Certain Pythagoreans¹¹³ had placed these dark planets between the earth and the central fire in order to account for their invisibility, and the next stage was to reduce them to a single body. Here again we see how the Pythagoreans tried to simplify the hypotheses of their predecessors.

152. The Harmony of the Spheres

We have seen (§ 54) that the doctrine commonly, but incorrectly, known as the "harmony of the spheres" may have originated with Pythagoras, but its elaboration must belong to a later generation, and the extraordinary variations in our accounts of it must be due to the conflicting theories of the planetary motions which were rife at the end of the fifth and the beginning of the fourth centuries B.C. We have the express testimony of Aristotle that the Pythagoreans whose doctrine he knew believed that the heavenly bodies produced musical notes in their courses. Further, the pitch of the notes was determined by the velocities of these bodies, and these in turn by their distances, which were in the same ratios as the consonant intervals of the octave. Aristotle distinctly implies that the heaven of the fixed stars takes part in the celestial symphony; for he mentions "the sun, the moon, and the stars, so great in magnitude and in number as they are," a phrase which cannot refer solely or chiefly to the five planets.¹¹⁴ We are also told that the slower bodies give out a deep note and the swifter a high note, and the prevailing tradition gives the high note of the octave to the heaven of the fixed stars, which revolves in twenty-four hours. Saturn, of course, comes next; for, though it has a slow motion of its own in a contrary direction, that is "mastered" (*ἡρατεῖται*) by the diurnal revolution. The other view, which gives the highest note to the Moon and the lowest to the fixed stars, is probably due to the theory which substituted an axial rotation of the earth for the diurnal revolution of the heavens.¹¹⁵

153. The Likenesses of Numbers

We have still to consider a view, which Aristotle sometimes attributes to the Pythagoreans, that things were "like numbers." He does not appear to regard this as inconsistent with the doctrine that things *are* numbers, though it is hard to see how he could reconcile the two.¹¹⁶ There is no doubt, however, that Aristoxenos represented the Pythagoreans as teaching that things were like numbers,¹¹⁷ and there are other traces of an attempt to make out that this was the original doctrine. A letter was

produced, purporting to be by Theano, the wife of Pythagoras, in which she says that she hears many of the Hellenes think Pythagoras said things were made *of* number, whereas he really said they were made *according to* number.¹¹⁸

When this view is uppermost in his mind, Aristotle seems to find only a verbal difference between Plato and the Pythagoreans. The metaphor of "participation" was merely substituted for that of "imitation." This is not the place to discuss the meaning of the so-called "theory of ideas"; but it must be pointed out that Aristotle's ascription of the doctrine of "imitation" to the Pythagoreans is abundantly justified by the *Phaedo*. When Simmias is asked whether he accepts the doctrine, he asks for no explanation of it, but replies at once and emphatically that he does. The view that the equal itself is alone real, and that what we call equal things are imperfect imitations of it, is quite familiar to him,¹¹⁹ and he is finally convinced of the immortality of the soul just because Sokrates makes him see that the theory of forms implies it.

It is also to be observed that Sokrates does not introduce the theory as a novelty. The reality of the "ideas" is the sort of reality "we are always talking about," and they are explained in a peculiar vocabulary which is represented as that of a school. The technical terms are introduced by such formulas as "we say."¹²⁰ Whose theory is it? It is usually supposed to be Plato's own, though some call it his "early theory of ideas," and say that he modified it profoundly in later life. But there are serious difficulties in this view. Plato is very careful to tell us that he was not present at the conversation recorded in the *Phaedo*. Did any philosopher ever propound a new theory of his own by representing it as already familiar to a number of distinguished living contemporaries?¹²¹ It is not easy to believe that. It would be rash, on the other hand, to ascribe the origin of the theory to Sokrates, and there seems nothing for it but to suppose that the doctrine of "forms" (εἶδη, ἰδέαι) originally took shape in Pythagorean circles, though it was further developed by Sokrates. There is nothing startling in this. It is a historical fact that Simmias and Kebes were not only Pythagoreans but disciples of Sokrates, and there were, no doubt, more "friends of the ideas"¹²² than we generally recognise. It is certain, in any case, that the use of the words εἶδη and ἰδέαι to express ultimate realities is pre-Platonic, and it seems most natural to regard it as of Pythagorean origin.

We have really exceeded the limits of this work by tracing the history of Pythagoreanism down to a point where it becomes practically indistinguishable from the theories which Plato puts into the mouth of Sokrates; but it was necessary to do so in order to put the statements of our authorities in their true light. Aristoxenos is not likely to have been mistaken with regard to the opinions of the men he had known personally, and Aristotle's statements must have had some foundation.

1. Iambl. *V. Pyth.* 251. The ultimate authority for all this is Timaios. There is no need to alter the MS. reading Ἀρχύτου to Ἀρχίππου (as Diels does after Beckmann). We are dealing with a later generation, and the sentence opens with of οἱ δὲ λοιποὶ τῶν Πυθαγορείων, *i.e.* those other than Archippos and Lysis, who have been dealt with in the preceding section.
2. For Philolaos, see Plato, *Phaed.* 61 d 7; e 7; and for Lysis, Aristoxenos in Iambl. *V. Pyth.* 250 (R. P. 59 b).
3. Diog. viii. 79-83 (R. P. 61). Aristoxenos himself came from Taras. The story of Damon and Phintias (told by Aristoxenos) belongs to this time.
4. Diog. viii, 46 (R. P. 62).
5. The whole *mise en scène* of the *Phaedo* presupposes this, and it is quite incredible that Plato should have misrepresented the matter. Simmias and Kebes were a little younger than Plato and he could hardly have ventured to introduce them as disciples of Sokrates if they had not in fact been so. Xenophon too (*Mem.* i. 2. 48) includes Simmias and Kebes in his list of genuine disciples of Sokrates, and in another place (iii. 11, 7) he tells us that they had been attracted from Thebes by Sokrates and never left his side.
6. See Aristoxenos *ap.* Val. Max. viii. 13, ext. 3 ; and Soudidas *s.v.*
7. See below, §§ 150-152.
8. Plato, *Phaed.* 61 d 6.
9. This appears to follow from the remark of Simmias in *Phaed.* 64 b. The whole passage would be pointless if the words φιλόσοφος, φιλοσοφεῖν, φιλοσοφία had not in some way become familiar to the ordinary Theban of the fifth century. Now Herakleides Pontikos made Pythagoras invent the word, and expound it in a conversation with Leon, tyrant of Sikyon or *Phleious*. Cf. Diog. i. 12 (R. P. 3), viii. 8; Cic. *Tusc.* v. 3. 8. Cf. also the remark of Alkidamas quoted by Arist. *Rhet.* B, 23. 1398 b i8, Θήβησιν ἅμα οἱ προστάται φιλόσοφοι ἐγένοντο καὶ εὐδαιμόνησεν ἡ πόλις.
10. For reasons which will appear, I do not attach importance in this connexion to Philolaos, fr. 14 Diels=23 Mullach (R. P. 89), but it does seem likely that the μυθολογῶν κομψὸς ἀνήρ of *Gorg.* 493 a 5 (R. P. 89 b) is responsible for the whole theory there given. He is certainly, in any case, the author of the τετρημένος πίθος, which implies the same general view. Now he is called ἴσως Σικελός τις ἢ Ἰταλικός, which means he was an Italian; for the Σικελός τις is merely an allusion to the Σικελός κομψὸς ἀνήρ ποτὶ τὰν ματέρ' ἔφα of Timokreon. We do not know of any Italian from whom Sokrates could have learnt these views except Philolaos or one of his associates.
11. See above, Chap. II. p. 102, *n.* 2.
12. It is a good illustration of the defective character of our tradition (Introd. p. 26) that this was quite unknown till the publication of the extracts from Menon's *Iatrika* contained in the *Anonymus Londinensis*. See Diels in *Hermes*, xxviii. pp. 417 *sqq.*
13. See p. 276, *n.* 2, and p. 278, *n.* 2.
14. This follows at once from the fact that he is represented as conversing with the elder Kritias (p. 203, *n.* 3), who is very aged, and with Hermokrates, who is quite young.
15. Diog. iii. 37. For similar charges, cf. Zeller, *Plato*, p. 429, *n.* 7.
16. Iambl. *V. Pyth.* 199. Diels is clearly right in ascribing the story to Aristoxenos (*Arch.* iii. p. 461, *n.* 26).
17. Timon, fr. 54 (Diels), *ap.* Gell. iii. 17 (R. P. 60 a).
18. For Hermippos and Satyros, see Diog. iii. 9; viii. 84, 85.
19. So Iambl. *in Nicom.* p. 105, 11; Proclus, *in Tim.* p. 1, Diehl.
20. They are τὰ θρυλούμενα τρία βιβλία (Iambl. *V. Pyth.* 199), τὰ διαβόητα τρία βιβλία (Diog. viii. 15).
21. As Bywater said (*J. Phil.* i. p. 29), the history of this work "reads like the history, not so much of a book, as of a literary *ignis fatuus* floating before the minds of imaginative writers."

22. Diels, "Ein gefälschtes Pythagorasbuch" (*Arch.* iii. pp. 451 *sqq.*).
23. Diog. viii. 85 (R. P. 63 b). Diels reads *πρῶτον ἐκδοῦναι τῶν Πυθαγορικῶν <βιβλία καὶ ἐπιγράψαι Περί> Φύσεως*.
24. Diog. viii. 7.
25. Proclus, *in Eucl.* p. 22, 15 (Friedlein). Cf. Boeckh, *Philolaos*, pp. 36 *sqq.* Boeckh refers to a sculptured group of *three* Bakchai, whom he supposes to be Ino, Agaue, and Autonoe.
26. The passage is given in R. P. 68. For a full discussion of this and the other fragments, see Bywater, "On the Fragments attributed to Philolaos the Pythagorean" (*J. Phil.* i. pp. 21 *sqq.*).
27. Boeckh, *Philolaos*, p. 38. Diels (*Vors.* p. 246) distinguishes the *Bakchai* from the three books *Περὶ φύσιος* (*ib.* p. 239). As, however, he identifies the latter with the "three books" bought from Philolaos, and regards it as genuine, this does not seriously affect the argument.
28. See Diels in *Arch.* iii. pp. 460 *sqq.*
29. On the Achaian dialect, see O. Hoffmann in Collitz and Bechtel, *Dialekt-Inschriften*, vol. ii. p. 151. How slowly Doric penetrated into the Chalkidian states may be seen from the mixed dialect of the inscription of Mikythos of Rhegion (*Dial.-Inschr.* iii. 2, p. 498), which is later than 468-67 B.C. There is no reason to suppose that the Achaian dialect of Kroton was less tenacious of life. We can see from Herodotos that there was a strong prejudice against the Dorians there.
30. The scanty fragments contain one Doric (or Achaian ?) form, *ἔχοντι* (fr. 1), but Alkmaion calls himself *Κροτωνιῆτης*, which is very significant; for *Κροτωνιάτας* is the Achaian as well as the Doric form.
31. *Arch.* iii. p. 460.
32. He is distinctly called a Krotoniate in the extracts from Menon's *Ἰατρικά* (cf. Diog. viii. 84). It is true that Aristoxenos called him and Eurytos Tarentines (Diog. viii. 46), but this only means that he settled at Taras after leaving Thebes. These variations are common in the case of migratory philosophers. Eurytos is also called a Krotoniate and a Metapontine (Iamb. *V. Pyth.* 148, 266). Cf. also p. 330, *n.* 1 on Leukippos, and p. 351, *n.* 1 on Hippon.
33. For Androkydes, see Diels, *Vors.* p. 281. As Diels points out (*Arch.* iii. p. 461), even Lucian has sufficient sense of style to make Pythagoras speak Ionic.
34. Cf. fr. 12=20 M. (R. P. 79), which I read as it stands in the MS. of Stobaios, but bracketing an obvious adscript or dittography, *καὶ τὰ ἐν τῷ σφαίρᾳ σώματα πέντε ἐντί [τὰ ἐν τῷ σφαίρᾳ], πῦρ, ὕδωρ καὶ γᾶ καὶ ἀήρ, καὶ ὁ τῶν σφαιρῶν ὀκτὰς πεμπτόν*. In any case, we are not justified in reading *τὰ μὲν τῶν σφαιρῶν σώματα* with Diels. For the identification of the four elements with four of the regular solids, cf. § 147, and for the description of the fifth, the dodecahedron, cf. § 148.
35. Plato, *Rep.* 528 b.
36. Heiberg's Euclid, vol. v. p. 654, 1, *ἐν τούτῳ τῷ βιβλίῳ, τουτέστι τῷ γ', γράφεται τὰ λεγόμενα Πλάτωνος ἐπιγράψαντων τῶν Πυθαγορείων ἐστίν, ὃ τε κύβος καὶ ἡ πυραμὶς καὶ τὸ δωδεκάεδρον, Θεαιτήτου δὲ τὸ τε ὀκτάεδρον καὶ τὸ εἰκοσάεδρον*. It is no objection to this that, as Newbold points out (*Arch.* xix. p. 204), the inscription of the dodecahedron is more difficult than that of the octahedron and icosahedron. We have no right to reject the definite testimony quoted above (no doubt from Eudemos) on grounds of *a priori* probability. As a matter of fact, there are Celtic and Etruscan dodecahedra of considerable antiquity in the Louvre and elsewhere (G. Loria, *Scienze esatte* p. 39), and the fact is significant in view of the connexion between Pythagoreanism and the North which has been suggested.
37. Philolaos is quoted only once in the Aristotelian corpus, in *Eth. Eud.* B, 8. 1225 a 33 *ἀλλ' ὥσπερ Φιλόλαος ἔφη εἶναί τινας λόγους κρείττους ἡμῶν*, which looks like an apophthegm. His name is not even mentioned anywhere else, and this would be inconceivable if Aristotle had ever seen a work of his which expounded the Pythagorean system. He must have known the importance of Philolaos from Plato's *Phaedo*, and would certainly have got hold of his book if it had existed. It should be added that Tannery held the musical theory of our fragments to be too advanced for Philolaos. It must, he argued, be later than Plato and Archytas (*Rev. de Phil.* xxviii. pp. 233 *sqq.*). His opinion on such a point is naturally of the greatest weight.
38. Aristotle says distinctly (*Met.* A, 6. 987 b 25) that "to set up a dyad instead of the unlimited regarded as one, and to make the unlimited consist of the great and small, is distinctive of Plato."
39. Zeller, p. 369 *sqq.* (Eng. trans. p. 397 *sqq.*).

40. For the doctrine of "Philolaos," cf. fr. 1 (R. P. 64); and for the unknowable ἐστὼ τῶν πραγμάτων, see fr. 3 (R. P. 67). It has a suspicious resemblance to the later ὕλη, which Aristotle would hardly have failed to note. He is always on the look-out for anticipations of ὕλη.
41. Arist. *Met.* A, 8. 989 b 29 (R. P. 92 a).
42. Arist. *Met.* A, 8. 990 a 3, ὁμολογοῦντες τοῖς ἄλλοις φυσιολόγοις ὅτι τό γ' ὄν τοῦτ' ἐστὶν ὅσον αἰσθητὸν ἐστὶ καὶ περιείληφεν ὁ καλούμενος οὐρανός.
43. Arist. *Met. ib.*, 8. 990 a 5, τὰς δ' αἰτίας καὶ τὰς ἀρχάς, ὥσπερ εἶπομεν, ἰκανὰς λέγουσιν ἐπαναβῆναι καὶ ἐπὶ τὰ ἀνωτέρω τῶν ὄντων, καὶ μᾶλλον ἢ τοῖς περὶ φύσεως λόγοις ἀρμοστούσας.
44. *Met.* A, 5. 986 a 1; τὰ τῶν ἀριθμῶν στοιχεῖα τῶν ὄντων στοιχεῖα πάντων ὑπέλαβον εἶναι N, 3. 1090 a 22 εἶναι μὲν ἀριθμοὺς ἐποίησαν τὰ ὄντα, οὐ χωριστοὺς δέ, ἀλλ' ἐξ ἀριθμῶν τὰ ὄντα.
45. *Met.* M, 6. 1080 b 2, ὡς ἐκ τῶν ἀριθμῶν ἐνυπαρχόντων ὄντα τὰ αἰσθητά; *ib.* 1080 b 17, ἐκ τούτου (τοῦ μαθηματικοῦ ἀριθμοῦ) τὰς αἰσθητὰς οὐσίας συνεστάναι φασίν.
46. *Met.* M, 8. 1083 b 11, τὰ σώματα ἐξ ἀριθμῶν εἶναι συγκείμενα; *ib.* b 17, ἐκείνοι δὲ τὸν ἀριθμὸν τὰ ὄντα λέγουσιν· τὰ γοῦν θεωρήματα προσάπτουσι τοῖς σώμασιν ὡς ἐξ ἐκείνων ὄντων τῶν ἀριθμῶν; N. 3. 1090 a 32 κατὰ μέντοι τὸ ποιεῖν ἐξ ἀριθμῶν τὰ φυσικὰ σώματα, ἐκ μὴ ἐχόντων βάρος μηδὲ κουφότητα ἔχοντα κουφότητα καὶ βάρος.
47. *Met.* A, 5. 986 a 2, τὸν ὅλον οὐρανὸν ἀρμονίαν εἶναι καὶ ἀριθμὸν; A, 8. 990 a 21 τὸν ἀριθμὸν τοῦτον ἐξ οὗ συνέστηκεν ὁ κόσμος; M. 6. 1080 b 18 τὸν γὰρ ὅλον οὐρανὸν κατασκευάζουσιν ἐξ ἀριθμῶν; *De caelo* Γ. 1. 300 a 15, τοῖς ἐξ ἀριθμῶν συνιστάσι τὸν οὐρανόν· ἔνιοι γὰρ τὴν φύσιν ἐξ ἀριθμῶν συνιστάσιν, ὥσπερ τῶν Πυθαγορείων τινές.
48. *Met.* N, 3. 1091 a 18, κοσμοποιούσι καὶ φυσικῶς βούλονται λέγειν.
49. *Met.* M, 6. 1080 b 16; N, 3. 1090 a 20.
50. Arist. *Met.* A, 5. 987 a 15.
51. *Met. ib.* 986 a 15 (R. P. 66).
52. *Met.* A, 6. 987 b 27, ὁ μὲν (Πλάτων) τοὺς ἀριθμοὺς παρὰ τὰ αἰσθητά, οἱ δ' (οἱ Πυθαγόρειοι) ἀριθμοὺς εἶναι φασὶν αὐτὰ τὰ αἰσθητά.
53. *Met.* A, 5. 986 a 17 (R. P. 66); *Phys.* Γ, 4. 203 a 10 (R. P. 66 a).
54. *Simpl. Phys.* p. 455, 20 (R. P. 66 a). I owe the passages which I have used in illustration of this subject to W. A. Heidel, "Πέρασ and ἄπειρον in the Pythagorean Philosophy" (*Arch.* xiv. pp. 384 *sqq.*). The general principle of my interpretation is the same as his, though I think that, by bringing the passage into connexion with the numerical figures, I have avoided the necessity of regarding the words ἢ γὰρ εἰς ἴσα καὶ ἡμίση διαίρεσις ἐπ' ἄπειρον as "an attempted elucidation added by Simplicius."
55. Aristoxenos, fr. 81, *ap.* Stob. i. p. 20, ἐκ τῶν Ἀριστοξένου Περὶ ἀριθμητικῆς. . . τῶν δὲ ἀριθμῶν ἄρτιοι μὲν εἰσὶν οἱ εἰς ἴσα διαιρούμενοι, περισσοὶ δὲ οἱ εἰς ἄνισα καὶ μέσον ἔχοντες.
56. [Plut.] *ap.* Stob. i. p. 22, 19, καὶ μὴν εἰς δύο διαιρουμένων ἴσα τοῦ μὲν περισσοῦ μονὰς ἐν μέσῳ περίεστι, τοῦ δὲ ἀρτίου κενὴ λείπεται χώρα καὶ ἀδέσποτος καὶ ἀνάριθμος, ὡς ἂν ἐνδεοῦς καὶ ἀτελοῦς ὄντος.
57. *Plut. De E apud Delphos*, 388 a, ταῖς γὰρ εἰς ἴσα τομαῖς τῶν ἀριθμῶν, ὁ μὲν ἄρτιος πάντη διῆσταμνος ὑπολείπει τινὰ δεκτικὴν ἀρχὴν οἷον ἐν ἑαυτῷ καὶ χώραν, ἐν δὲ τῷ περιττῷ ταῦτο παθόντι μέσον αἰεὶ περίεστι τῆς νεμήσεως γόνιμον. The words which I have omitted in translating refer to the further identification of Odd and Even with Male and Female. The passages quoted by Heidel might be added to. Cf., for instance, what Nikomachos says (p. 13, 10, Hoche), ἔστι δὲ ἄρτιον μὲν ὁ οἶόν τε εἰς δύο ἴσα διαιρεθῆναι μονάδος μέσον μὴ παρεμπιπτούσης, περιττὸν δὲ τὸ μὴ δυνάμενον εἰς δύο ἴσα μερισθῆναι διὰ τὴν προειρημένην τῆς μονάδος μεστέϊαν. He significantly adds that this definition is ἐκ τῆς δημόδους ὑπολήψεως.

58. Arist. *Phys.* Γ, 4. 204 a 20 *sqq.*, especially a 26, ἀλλὰ μὴν ὥσπερ ἀέρος ἀήρ μέρος, οὕτω καὶ ἄπειρον ἀπέιρου, εἴ γε οὐσία ἐστὶ καὶ ἀρχή.
59. See Chap. II. § 53.
60. Ar. *Phys.* Δ, 9. 216 b 25, κυμανεῖ τὸ ὄλον.
61. Cf. Speusippos in the extract preserved in the *Theologumena arithmetica*, p. 61 (Diels, *Vors.* 32 A 13) τὸ μὴν γὰρ [α] στιγμῆ, τὰ δὲ [β] γραμμῆ, τὰ δὲ [γ] τρίγωνον, τὰ δὲ [δ] πυραμίδς. We know that Speusippos is following Philolaos here. Arist. *Met.* Z, 11. 1036 b 12, καὶ ἀνάγουσι πάντα εἰς τοὺς ἀριθμούς, καὶ γραμμῆς τὸν λόγον τὸν τῶν δύο εἶναι φασιν.. The matter is clearly put by Proclus in *Eucl. I.* p. 97, 19, τὸ μὲν σημεῖον ἀνάλογον τίθενται μονάδι, τὴν δὲ γραμμὴν δυάδι, τὴν δὲ ἐπιφάνειαν τῆ τριάδι καὶ τὸ στερεὸν τῆ τετράδι. καίτοι γε ὡς διαστατὰ λαμβάνοντες μοναδικὴν μὲν εὐρήσομεν τὴν γραμμὴν, δυαδικὴν δὲ τὴν ἐπιφάνειαν, τριαδικὸν δὲ τὸ στερεόν.
62. The identification of the point with the unit is referred to by Aristotle, *Phys.* B. 227 a 27.
63. Arist. *Met.* M, 6. 1080 b 18 *sqq.*, 1083 b 8 *sqq.* ; *De caelo*, Γ, 1. 300 a 16 (R. P. 76 a).
64. Zeller, p. 381.
65. Zeno in his fourth argument about motion, which, we shall see (§ 163), was directed against the Pythagoreans, used ὄγκοι for points. Aetios, i. 3, 19 (R. P. 76 b), says that Ekphantos of Syracuse was the first of the Pythagoreans to say that their units were corporeal. Cf. also the use of
- ὄγκοι in Plato, *Parm.* 164 d, and Galen, *Hist. Phil.* 18 (*Dox.* p. 610), Ἡρακλείδης δὲ ὁ Ποντικὸς καὶ Ἀσκληπιάδης ὁ βιθυνὸς ἀνάγκους ὄγκους τὰς ἀρχὰς ὑποτίθενται τῶν ὄλων.
66. Zeller, p. 381.
67. Arist. *Met.* A, 8.,990 a 22 (R. P. 81 e). I read and interpret thus "For, seeing that, according to them, Opinion and Opportunity are in a given part of the world, and a little above or below them Injustice and Separation and Mixture,—in proof of which they allege that each of these is a number,—and seeing that it is also the case (reading συμβαίνῃ with Bonitz) that there is already in that part of the world a number of composite magnitudes (*i.e.* composed of the Limit and the Unlimited), because those affections (of number) are attached to their respective regions (seeing that they hold these two things), the question arises whether the number which we are to understand each of these things (Opinion, etc.) to be is the same as the number in the world (*i.e.* the cosmological number) or a different one." I cannot doubt that these are the extended numbers which are composed (συνίσταται) of the elements of number, the limited and the unlimited, or, as Aristotle here says, the "affections of number," the odd and the even. Zeller's view that "celestial bodies" are meant comes near this, but the application is too narrow. Nor is it the number (πλήθος) of those bodies that is in question, but their magnitude (μέγεθος). For other views of the passage see Zeller, p. 391, *n.* 1.
68. All this has been put in its true light by the publication of the extract from Menon's *Ἰατρικά* on which see p. 278, *n.* 4.
69. In Aet. ii. 6, 5 (R. P. 80) the theory is ascribed to Pythagoras, which is an anachronism, as the mention of "elements" shows it must be later than Empedokles. In his extract from the same source, Achilles says οἱ Πυθαγόρειοι which doubtless represents Theophrastos better.
70. See above p. 283.
71. Plato, *Tim.* 31 b 5.
72. Plato, *Tim.* 54 c 4. It is to be observed that in *Tim.* 48 b 5 Plato says of the construction of the elements οὐδεὶς πῶ γένεσιν αὐτῶν μεμήνηκεν, which implies that there is some novelty in the theory as Timaios states it. If we read the passage in the light of what has been said in § 141, we shall be inclined to believe that Plato is making Timaios work out the Pythagorean doctrine on the lines of the discovery of Theaitetos.
73. See above, Chap. IV. p. 186.
74. Aet. ii. 6, 5 (R. P. 80) ; "Philolaos," fr. 12 (=20 M.; R. P. 79). On the ὀλκάς, see Gundermann in *Rhein. Mus.* 1904, pp. 145 *sqq.* In the Pythagorean myth of Plato's *Politicus*, the world is regarded as a ship, of which God is the κυβερνήτης (272 a *sqq.*). The πόντος τῆς ἀνομοιότητος (273 d) is just the ἄπειρον.

75. Aet. ii. 4, 15, ὅπερ τρόπεως δίκην προὔπεβάλετο τῆ τοῦ παντός <σφαίρα> ὁ δημιουργὸς θεός.
76. Cf. the ὑποζώματα of Plato, *Rep.* 616 c 3. As ὕλη generally means "timber" for shipbuilding (when it does not mean firewood), I suggest that we should look in this direction for an explanation of the technical use of the word in later philosophy. Cf. Plato, *Phileb.* 54 c 1, γενέσεως . . . ἔνεκα . . . πᾶσαν ὕλην παρατίθεσθαι πᾶσιν, which is part of the answer to the question πότερα πλοίων ναυπηγίαν ἔνεκα φῆς γίνεσθαι μᾶλλον ἢ πλοία ἔνεκα ναυπηγίας; (*ib.* b 2); *Tim.* 69 a 6, οἷα τέκτοσιν ἡμῖν ὕλη παράκειται.
77. Plato, *Phaed.* 110 b 6, ὥσπερ οἱ δωδεκάσκυτοι σφαῖραι, the meaning of which phrase is quite correctly explained by Plutarch, *Plat. q.* 1003 b καὶ γὰρ μάλιστα τῷ πλήθει τῶν στοιχείων ἀμβλύτητι δὲ τῶν γωνιῶν τὴν εὐθύτητα διαφυγὸν εὐκαμπές ἐστι [τὸ δωδεκάεδρον], καὶ τῆ περιτάσει ὥσπερ αἱ δωδεκάσκυτοι σφαῖρα κυκλοτερές γίνεται καὶ περιληπτικόν.
78. Plato, *Tim.* 55 c 4. Neither this passage nor the last can refer to the Zodiac, which would be described by a dodecagon, not a dodecahedron. What is implied is the division of the heavens into twelve pentagonal fields, in which the constellations were placed. For the history of such methods see Newbold in *Arch.* xix. pp. 198 *sqq.*
79. Iambl. *V. Pyth.* 247. Cf. above, Chap. II. p.106, n. 1.
80. See Gow, *Short History of Greek Mathematics*, p. 151, and the passages there referred to, adding Schol. Luc. p. 234, 21, Rabe, τὸ πεντάγραμμον] ὅτι τὸ ἐν τῇ συνηθείᾳ λεγόμενον πεντάλφα σύμβολον ἦν πρὸς ἀλλήλους Πυθαγορείων ἀναγνωριστικὸν καὶ τούτω ἐν ταῖς ἐπιστολαῖς ἐχρῶντο. The Pythagoreans may quite well have known the method given by Euclid iv. 11 of dividing a line in extreme and mean ratio, the so-called "golden section."
81. Arist. *De an.* A, 3. 407 b 20 (R. P. 86 c).
82. Plato, *Phaed.* 85 e *sqq.*; and for Echekrates, *ib.* 88 d.
83. Plato, *Phaed.* 86 b7-c5.
84. See J. L. Stocks, *Plato and the Tripartite Soul* (Mind N.S., No. 94, 1915, pp. 207 *sqq.*). Plato himself points to the connexion in *Rep.* 443 d, 5 συναρμόσαντα τρία ὄντα, ὥσπερ ὄρους τρεῖς ἀρμονίας ἀτεχνῶς, νεάτης τε καὶ ὑπάτης καὶ μέσης, καὶ εἰ ἄλλα ἄττα μεταξὺ τυγχάνει ὄντα (*i.e.* the movable notes). Now there is good ground for believing that the statement of Aristides Quintilianus (ii. 2) that the θυμικόν is intermediate between the λογικόν and the ἄλογον comes from the musician Damon (Deiters, *De Aristidis Quinti. fontibus*, 1870), the teacher of Perikles (p. 255, n. 2), to whom the Platonic Sokrates refers as his authority on musical matters, but who must have died when Plato was quite young. Moreover, Poseidonios (ap. Galen, *De Hipp. et Plat.* pp. 425 and 478) attributed the doctrine of the tripartite soul to Pythagoras, αὐτοῦ μὲν τοῦ Πυθαγόρου συγγράμματος οὐδενὸς εἰς ἡμᾶς διασφραζομένου, τεκμαίρομενος δὲ ἐξ ὧν ἔνιοι τῶν μαθητῶν αὐτοῦ γεγράφασιν.
85. For the authorities see R. P. 81-83. The attribution of the theory to Philolaos is perhaps due to Poseidonios. The "three books" were doubtless in existence by his time.
86. Plato makes Timaios attribute an axial rotation to the heavenly bodies, which must be of this kind (*Tim.* 40 a 7). The rotation of the moon upon its axis takes the same time as its revolution round the earth; but it comes to the same thing if we say that it does not rotate at all relatively to its orbit, and that is how the Greeks put it. It would be quite natural for the Pythagoreans to extend this to all the heavenly bodies. This led ultimately to Aristotle's view that they were all fixed (ἐνδεδεμένα) in corporeal spheres.
87. This seems more natural than to suppose the earth and counter-earth to be always in conjunction. Cf. Aet. iii. 11, 3, τὴν οἰκουμένην γῆν ἐξ ἐναντίας κεκμένην καὶ περιφερομένην τῆ ἀντίχθονι.
88. Plato, *Phaed.* 108 e 4 *sqq.* Simmias assents to the geocentric theory in the emphatic words καὶ ὀρθῶς γε.
89. Aet. ii. 20, 13 (Chap. VI. p. 238, n. 3) compared with *ib.* 12 Φιλόλαος ὁ Πυθαγόρειος ὑαλοειδῆ τὸν ἥλιον, δεχόμενον μὲν τοῦ ἐν τῷ κόσμῳ πυρὸς τὴν ἀνατύγειαν, διηθούντα δὲ πρὸς ἡμᾶς τὸ φῶς, ὥστε τρόπον τινὰ διττοὺς ἡλίους γίνεσθαι, τό τε ἐν τῷ οὐρανῷ πυρῶδες καὶ τὸ ἀπ' αὐτοῦ πυροειδὲς κατὰ τὸ ἐσοπτροειδές· εἰ μὴ τις καὶ τρίτον λέξει τὴν ἀπὸ τοῦ ἐνόπτρου κατ' ἀνάκλασιν διασπειρομένην πρὸς ἡμᾶς αὐγὴν. This is not, of course, a statement of any doctrine held by "Philolaos," but a rather captious criticism such as we often find in Theophrastos. Moreover, it is pretty clear that it is inaccurately reported. The phrase τὸ ἐν τῷ κόσμῳ πῦρ, if used by Theophrastos, must surely mean the central fire and τὸ ἐν τῷ οὐρανῷ πυρῶδες must be the same thing, as it very well may, seeing that Aetios tells us himself (ii. 7. 7, R. P. 81) that "Philolaos" used the term οὐρανός of the sublunary region. It is true that Achilles says τὸ πυρῶδες καὶ διαυγὲς λαμβάνοντα ἄνωθεν ἀπὸ τοῦ ἀερίου πυρός, but his authority is not sufficiently great to outweigh the other considerations.

90. Aet. i. 7, 7 (R. P. 81). Proclus in *Tim.* p. 106, 22 (R. P. 83 e).

91. Aristotle expresses this by saying that the Pythagoreans held τὴν . . . γῆν ἐν τῶν ἄστρον οὐσαν κύκλω φερομένην περὶ τὸ μέσον νύκτα τε καὶ ἡμέραν ποιεῖν (*De caelo*, B, 13. 293 a 23).

92. I do not discuss here the claims of Herakleides to be the real author of the heliocentric hypothesis.

93. In a letter to Pope Paul III., Copernicus quotes Plut. *Plac.* iii. 13, 2-3 (R. P. 83 a) and adds *Inde igitur occasionem nactus, coepi et ego de terrae mobilitate cogitare.*

94. Cf. Ar. *De caelo*, B, 13. 293 b 25 ἐπεὶ γὰρ οὐκ ἔστιν ἡ γῆ κέντρον, ἀλλ' ἀπέχει τὸ ἡμισφαίριον αὐτῆς ὅλον, οὐθὲν κωλύειν οἴονται τὰ φαινόμενα συμβαίνειν ὁμοίως μὴ κατοικοῦσιν ἡμῖν ἐπὶ τοῦ κέντρου, ὥσπερ κὰν εἰ ἐπὶ τοῦ μέσου ἦν ἡ γῆ· οὐθὲν γὰρ οὐδὲ νῦν ποιεῖν ἐπίδηλον τὴν ἡμισείαν ἀπέχοντας ἡμᾶς διάμετρον. (Of course the words τὸ ἡμισφαίριον αὐτῆς ὅλον refer to Aristotle's own theory of celestial spheres; he really means the radius of its orbit.) Now it is inconceivable that any one should have argued that, since the geocentric parallax is negligible, parallax in general is negligible. On the other hand, the geocentric Pythagorean (the real Philolaos?), whose views are expounded by Sokrates in the *Phaedo*, appears to have made a special point of saying that the earth was πάμμεγα (109 a 9), and that would make the theory of the central fire very difficult to defend. If Philolaos was one of the Pythagoreans who held that the radius of the moon's orbit is only three times that of the earth's (Plut. *De an. procr.* 1028 b), he cannot have used the argument quoted by Aristotle.

95. Aet. iii. 13, 3 Ἡρακλείδης ὁ Ποντικός καὶ Ἐκφάντος ὁ Πυθαγόρειος κινουσί μὲν τὴν γῆν· οὐ μὴν γε μεταβατικῶς, ἀλλὰ τρεπτικῶς [1. στρεπτικῶς] τρόχου δίκην ἐνηξονισμένην, ἀπὸ δυσμῶν ἐπ' ἀνατολὰς περὶ τὸ ἴδιον αὐτῆς κέντρον. Cicero attributes the same doctrine to Hiketas (*Acad. pr.* ii. 39), but makes nonsense of it by saying that he made the sun and moon stationary as well as the fixed stars. Tannery regarded Hiketas and Ekphantos as fictitious personages from a dialogue of Herakleides, but it seems clear that Theophrastos recognised their existence. It may be added that the idea of the earth's rotation was no novelty. The Milesians probably (§ 21) and Anaxagoras certainly (p. 269) held this view of their flat earth. All that was new was the application of it to a sphere. If we could be sure that the geocentric Pythagoreans who made the earth rotate placed the central fire in the interior of the earth, that would prove them to be later in date than the system of "Philolaos." Simplicius appears to say this (*De caelo*, p. 512 9 *sqq.*), and he may be quoting from Aristotle's lost work on the Pythagoreans. The point, however, is doubtful.

96. The various possibilities are enumerated by Sir T. L. Heath (*Aristarchus*, p. 103). Only two are worth noting. The universe as a whole might share in the rotation of the ἀπλανές, while the sun, moon and planets had independent revolutions in addition to that of the universe. Or the rotation of the ἀπλανές might be so slow as to be imperceptible, in which case its motion, "though it is not the precession of the equinoxes, is something very like it" (Heath, *loc. cit.*).

97. Arist. *De caelo*, B, 13. 293 b 5, ἔνιοι δὲ καὶ κειμένην ἐπὶ τοῦ κέντρου [τὴν γῆν] φασὶν αὐτὴν ἵλλεσθαι καὶ κινεῖσθαι περὶ τὸν διὰ παντὸς τεταμένον πόλον, ὥσπερ ἐν τῷ Τιμαίῳ γεγραπταί. The text and interpretation of this passage are guaranteed by the reference in the next chapter (296 a 25) οἱ δ' ἐπὶ τοῦ μέσου θέντες ἵλλεσθαι καὶ κινεῖσθαι φασὶ περὶ τὸν πόλον μέσον. All attempts to show that this refers to something else are futile. We cannot, therefore, with Alexander, regard καὶ κινεῖσθαι as an interpolation in the first passage, even though it is omitted in some MSS. there. The omission is probably due to Alexander's authority. Moreover, when read in its context, it is quite clear that the passage gives one of two alternative theories of the earth's motion, and that this motion, like the revolution round the central fire, is a motion of translation (φορά), and not an axial rotation.

98. Plato's Doctrine respecting the Rotation of the Earth (1860).

99. Plato, *Tim.* 39 c 1, νῦξ μὲν οὖν ἡμέρα τε γέγονεν οὕτως καὶ διὰ ταῦτα, ἡ τῆς μιᾶς καὶ φρονιμωτάτης κυκλήσεως περίοδος. This refers to the revolution of the "circle of the Same," *i.e.* the equatorial circle, and is quite unambiguous.

100. Plato, *Tim.* 40 c 1 [γῆν] φύλακα καὶ δημιουργὸν νυκτός τε καὶ ἡμέρας ἐμχανήσατο. On this cf. Heath, *Aristarchus*, p. 178.

101. Arist. *De caelo*, B, 14. 296 a 29 *sqq.* The use of the word ὑπολειπόμενα of the apparent motion of the planets from west to east is an interesting survival of the old Ionian view (p. 70). The idea that the earth must have two motions, if it has any, is based on nothing more than the analogy of the planets (Heath, *Aristarchus*, p. 241).

102. Aristotle must have been a member of the Academy when the *Timaeus* was published, and we know that the interpretation of that dialogue was one of the chief occupations of the Academy after Plato's death. If he had misrepresented the doctrine by introducing a motion of translation, Alexander and Simplicius would surely have been able to appeal to an authoritative protest by Krantor or another. The view which Boeckh finds in the *Timaeus* is precisely Aristotle's own, and it is impossible to believe that he could have failed to recognise the fact or that he should have misrepresented it deliberately.

103. The best attested reading in *Tim.* is γῆν δὲ τροφὸν μὲν ἡμετέραν, ἰλλομένην δὲ τὴν περὶ τὸν διὰ παντὸς πόλον τεταμένον. The article τὴν is in Par. A and also in the Palatine excerpts, and it is difficult to suppose that any one would interpolate it. On the other

hand, it might easily be dropped, as its meaning is not at once obvious. It is to be explained, of course, like τὴν ἐπὶ θάνατον or Xenophon's προεληλυθότος . . . τὴν πρὸς τὰ φρούρια, and implies a *path* of some kind, and therefore a movement of translation.

104. In the first place, the meaning *globatam*, "packed," "massed" would have to be expressed by a perfect participle and not a present, and we find accordingly that Simplicius is obliged to paraphrase it by the perfect participle, δεδεμένη or δεδεσμημένη. Sir T. L. Heath's "wound" (*Aristarchus*, p. 177) ought also to be "winding." In the second place, though Par. A has εἰλλομένην, the weight of authority distinctly favours ἰλλομένην, the reading of Aristotle, Proclus and others. The verbs εἶλλω (εἶλλω), εἰλῶ and ἰλλω are constantly confused in MSS. It is not, I think, possible to regard ἰλλω as etymologically connected with the other verbs. It seems rather to go with ἰλλός and ἰλλαίνω, which are both used in Hippocrates. For its meaning, see below, *n. 2*.

105. Cf. Soph. *Ant.* 340 ἰλλομένων ἀρότρων ἔτος εἰς ἔτος, clearly of the ploughs going backwards and forwards in the furrows. Simplicius makes a point of the fact that Apollonios Rhodios used ἰλλόμενος in the sense of "shut in," "bound," εἰργόμενος (cf. Heath, *Aristarchus*, p. 175, *n. 6*). That, however, cannot weigh against the probability that the scribes, or even Apollonios himself, merely fell into the common confusion. Unless we can get rid of the article τὴν and the testimony of Aristotle, we must have a verb of motion.

106. Cf. Plato, *Phaed.* 111 c 4, where we are told that there is an αἰώρα in the earth, which causes the waters to move up and down in Tartaros, which is a chasm extending from pole to pole. See my notes *in loc.*

107. Proclus, in his commentary, explains the προχωρήσεις and ἐπανακυκλήσεις Of *Tim.* 20 c as equivalent to προποδισμοί and ὑποποδισμοί. In a *corrigendum* prefixed to his *Aristarchus*, Sir T. L. Heath disputes this interpretation, and compares the application of the term ἐπανακυκλούμενον to the planet Mars in *Rep.* 617 b, which he understands to refer merely to its "circular revolution in a sense contrary to that of the fixed stars." It is to be observed, however, that Theon of Smyrna in quoting this passage has the words μάλιστα τῶν ἄλλων after ἐπανακυκλούμενον, which gives excellent sense if retrogradation is meant. In fact Mars has a greater arc of retrogradation than the other planets (Duhem, *Système du monde*, vol. i. p. 61). As I failed to note this in my text of the *Republic*, I should like to make amends by giving two reasons for believing that Theon has preserved Plato's own words. In the first place he is apparently quoting from Derkyllides, who first established the text of Plato from which ours is derived. In the second place, μάλιστα τῶν ἄλλων is exactly fifteen letters, the normal length of omissions in the Platonic text.

108. Plut. *Plat. quaest.* 1006 c (cf. *V. Numa*, c. 11). It is important to remember that Theophrastos was a member of the Academy in Plato's last years.

109. In the passage referred to (822 a 4 *sqq.*) he maintains that the planets have a simple circular motion, and says that this is a view which he had not heard in his youth nor long before. That must imply the rotation of the earth on its axis in twenty-four hours, since it is a denial of the Pythagorean theory that the planetary motions are composite. It does not follow that we must find this view in the *Timaeus*, which only professes to give the opinions of a fifth-century Pythagorean.

110. Arist. *Met.* A, 5. 986 a 3 (R. P. 83 b).

111. Aet. ii. 29, 4, τῶν Πυθαγορείων τινὲς κατὰ τὴν Ἀριστοτέλειον ἱστορίαν καὶ τὴν Φιλίππου τοῦ Ὀπουντίου ἀπόφασιν ἀνταγορεύει καὶ ἀντιφράζει τοτὲ μὲν τῆς γῆς, τοτὲ δὲ τῆς ἀντίχθονος (ἐκλείπειν τὴν σελήνην).

112. Arist. *De caelo*, B, 13. 293 b 21, ἐνίοις δὲ δοκεῖ καὶ πλείω σώματα τοιαῦτα ἐνδέχασθαι φέρεσθαι περὶ τὸ μέσον ἡμῖν ἄδηλα διὰ τὴν ἐπιπρόσθησιν τῆς γῆς. διὸ καὶ τὰς τῆς σελήνης ἐκλειψεις πλείους ἢ τὰς τοῦ ἡλίου γίνεσθαι φασιν· τῶν γὰρ φερομένων ἕκαστον ἀντιφράττειν αὐτὴν, ἀλλ' οὐ μόνον τὴν γῆν.

113. It is not expressly stated that they were Pythagoreans, but it is natural to suppose so. So, at least, Alexander thought (*Simpl. De caelo*, p. 515, 25).

114. Arist. *De caelo*, B, 9. 290 b, 12 *sqq.* (R. P. 82). Cf. Alexander, *In met.* p. 39, 24 (from Aristotle's work on the Pythagoreans) τῶν γὰρ σωμάτων τῶν περὶ τὸ μέσον φερομένων ἐν ἀναλογία τὰς ἀποστάσεις ἐχόντων . . . ποιούντων δὲ καὶ ψόφον ἐν τῷ κινεῖσθαι τῶν μὲν βραδυτέρων βαρύν, τῶν δὲ ταχύτερων ὀξύν. There are all sorts of difficulties in detail. We can hardly attribute the identification of the seven planets (including sun and moon) with the strings of the heptachord to the Pythagoreans of this date; for Mercury and Venus have the same mean angular velocity as the Sun, and we must take in the heaven of the fixed stars.

115. For the various systems, see Boeckh, *Kleine Schriften*, vol. iii. pp. 169 *sqq.*, and Carl v. Jan, "Die Harmonie der Sphären" (*Philol.* 1893. pp. 13 *sqq.*). There is a sufficient account of them in Heath's *Aristarchus*, pp. 107 *sqq.*, where the distinction between absolute and relative velocity is clearly stated, a distinction which is not appreciated in Adam's note on *Rep.* 617 b (vol. ii. p. 452), with the result that, while the heaven of the fixed stars is rightly regarded as the νῆτη (the highest note), the Moon comes next instead of Saturn—an impossible arrangement. The later view is represented by the "bass of Heaven's deep Organ" in the "ninefold harmony" of Milton's *Hymn on the Nativity* (xiii.). At the beginning of the Fifth Act of the *Merchant of Venice*, Shakespeare makes

Lorenzo expound the doctrine in a truly Pythagorean fashion. According to him, the "harmony" in the soul ought to correspond with that of the heavenly bodies ("such harmony is in immortal souls"), but the "muddy vesture of decay" prevents their complete correspondence. The *Timaeus* states a similar view, and in the *Book of Homage to Shakespeare* (pp. 58 sqq.) I have tried to show how the theories of the *Timaeus* may have reached Shakespeare. There is no force in Martin's observation that the sounding of all the notes of an octave at once would not produce a harmony. There is no question of harmony in the modern sense, but only of attunement (ἄρμονία) to a perfect scale.

116. Cf. especially *Met.* A, 6. 787, b 10 (R. P. 65 d). It is not quite the same thing when he says, as in A, 5. 985 b 23 sqq. (R. P. *ib.*), that they perceived many likenesses in things to numbers. That refers to the numerical analogies of justice, Opportunity, etc.

117. Aristoxenos *ap.* Stob. i. pr. 6 (p. 20), Πυθαγόρας . . . πάντα τὰ πράγματα ἀπεικάζων τοῖς ἀριθμοῖς.

118. Stob. *Ecl.* i. p. 125, 19 (R. P. 65 d).

119. Plato, *Phaed.* 74 a sqq.

120. Cf. especially the words ὁ θρυλοῦμεν αἰεί (76 d 8). The phrases αὐτὸ ὃ ἔστιν, αὐτὸ καθ' αὐτό, and the like are assumed to be familiar. "We" define reality by means of question and answer, in the course of which "we" give an account of its being (ἡς λόγον δίδομεν τοῦ εἶναι, 78 d 1, where λόγον . . . τοῦ εἶναι is equivalent to λόγον τῆς οὐσίας). When we have done this, "we" set the seal or stamp of αὐτὸ ὃ ἔστιν upon it (75 d 2). Technical terminology implies a school. As Diels puts it (*Elementum*, p. 20), it is in a school that "the simile concentrates into a metaphor, and the metaphor condenses into a term."

121. In the *Parmenides* Plato makes Sokrates expound the theory at a date which is carefully marked as at least twenty years before his own birth.

122. Plato, *Soph.* 248 a 4. Proclus says (*in Parm.* iv. p. 149, Cousin) ἦν μὲν γὰρ καὶ παρὰ τοῖς Πυθαγορείοις ἡ περὶ τῶν εἰδῶν θεωρία, καὶ δηλοῖ καὶ αὐτὸς ἐν Σοφιστῇ τῶν εἰδῶν φίλους προσαγορεύων τοὺς ἐν Ἰταλίᾳ σοφοὺς, ἀλλ' ὁ γε μάλιστα πρεσβεύσας καὶ διαρρήδην ὑποθέμενος τὰ εἶδη Σωκράτης ἐστίν. This is not in itself authoritative; but it is the only statement on the subject that has come down to us, and Proclus (who had the tradition of the Academy at his command) does not appear to have heard of any other interpretation of the phrase. In a later passage (v. p. 4, Cousin) he says it was natural for Parmenides to ask Sokrates whether he had thought of the theory for himself, since he might have heard a report of it.